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Integrating System-Level and Component-Level Designs Under Uncertainty

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A probabilistic optimization methodology for aerospace vehicle design is developed that takes into account coupling between system-level and component-level design requirements. The proposed methodology represents the system-component design as a reliability-based multidisciplinary optimization problem. The system design optimizes the geometry of a reusable launch vehicle for minimum weight while satisfying aerodynamic constraints. The component design relates to the structural sizing of vehicle components, in this case a liquid-hydrogen tank. The optimization formulation includes probabilistic constraints, which are evaluated using the limit state-based reliability analysis methodology. The system and component designs are linked through stochastic state variables relating to vehicle geometry and component weight, and the optimization at both levels is pursued. Finally, the reliability-based multidisciplinary optimization is solved using an efficient decoupled approach.

Nomenclature

A	=	aerodynamic analysis ft/s^2 (m/s^2)
a	=	acceleration, ft/s^2
aoa	=	angle of attack, deg
bfar	=	body flap area ratio, ft^2/ft^2 (m^2/m^2)
bflap	=	body flap deflection, deg
bl	=	ballast weight fraction, lb/lb (kg/kg)
C_L	=	coefficient of lift
C_m	=	pitching-moment coefficient
delev	=	elevon deflection, deg
f	=	probability distribution function
fr	=	fuselage finess ratio, ft/ft (m/m)
g	=	limit state function
g_{HCB}	=	limit state function for honeycomb buckling failure
g_{ISO}	=	limit state function for isotropic strength failure
g_{VM}	=	limit state function for Von Mises failure
L/D	=	hypersonic lift-to-drag ratio
mr	=	mass ratio, lb/lb (kg/kg)
P	=	probability
P_f	=	probability of failure
$\boldsymbol{R}$	=	location of reactions from simple beam analysis, in. (cm)
$\mathrm{rf}_{\mathrm{tank\ weight}}$	=	tank weight reduction factor
S	=	standard reference wing area, ft^2
T	=	tank sizing analysis
t_{hc}	=	thickness of honeycomb material in tank wall, in. (cm)
t_{plate}	=	thickness of plate material in honeycomb sandwich tank wall, in. (cm)
tfar	=	tip fin area ratio, ft^2/ft^2 (m^2/m^2)
tvc	=	tail volume coefficient
u	=	multidisciplinary state variables, that is, output from one discipline and input to another
u_{WA}	=	state variables passing from weights analysis W to aerodynamic analysis A
u_{WT}	=	state variables passing from weights analysis W to tank sizing analysis T
W	=	weights analysis
W_{empty}	=	vehicle empty (dry) weight, lb
W_i	=	predicted weight of ith component, lb
war	=	wing area ratio, ft^2/ft^2 (m^2/m^2)
X	=	optimization design variables
$\boldsymbol{X}_{\mathrm{AeroControl}}$	=	vector of aerodynamic design variables (delev and bflap for each of nine scenarios)
$\boldsymbol{X}_{\mathrm{Geometry}}$	=	vector of geometry design variables (fr, war, tfar, bfar, mr, bl)
$\boldsymbol{X}_{\mathrm{System}}$	=	vector of output variables from system design needed for tank design
$\boldsymbol{x}^*$	=	performance-measure-approach (PMA) design point
$\lvert\boldsymbol{x}\rvert_2$	=	l_2 norm of vector $\boldsymbol{x}$
Y	=	nondesign random input variables
$\boldsymbol{Y}_{\mathrm{AeroControl}}$	=	vector of random (nondesign) aerodynamic variables angle of attack for each of nine scenarios)
$\boldsymbol{Y}_{\mathrm{Mission}}$	=	vector of random (nondesign) mission variables ($\boldsymbol{R}$, $\boldsymbol{a}$, % fuel, mix ratio, t_{hc})
β	=	reliability index
$\boldsymbol{\eta}$	=	vector of variables in standard normal space
η^*	=	PMA design point in standard normal space
μ	=	mean value
μ^N	=	equivalent normal mean value
ρ_{plate}	=	density of plate material used in honeycomb sandwich tank wall
ρ_{hc}	=	density of honeycomb material used in honeycomb sandwich tank wall
σ	=	standard deviation
σ^N	=	equivalent normal standard deviation
Φ	=	standard normal cumulative distribution function
∇	=	gradient

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Introduction

TODAY'S aerospace industry is faced with the challenge of engineering complex systems for which cost effectiveness and reliability are given the highest priority. Consider for example two goals for next-generation reusable launch vehicles (RLV): 1) a $1000/lb ($2205/kg) or less cost for payload delivery and 2) a less than 1/10,000 risk of crew loss.[1] This will require significant improvement (10-fold and 100-fold, respectively) over the current generation. Whether these targets are achievable remains to be seen, but it is clear that a systematic design approach is needed to move toward such targets. Uncertainties in system characteristics and demand, however, complicate the design process. These uncertainties prevent assurances of system performance from being given with absolute confidence. Thus system assessment, an integral part of the design process, must include measures of performance uncertainty. Using these measures to constrain design optimization is commonly referred to as reliability-based design optimization (RBDO). In particular, this paper addresses how RBDO can be used to couple two levels of design under uncertainty.

Systems engineering uses a design approach driven by top-level requirements.[2] However, design assessment is typically facilitated by discipline-specific (e.g., structural, aerodynamic, etc.) computational analysis models. These discipline models must be integrated into a multidisciplinary analysis before designs can be evaluated at the system level. From this point a top-down design approach can be undertaken. At the higher levels of design, more of the system is considered with less detail. As the design process continues, smaller components are designed to a greater level of detail. The highest (system or conceptual) level design provides the framework for design at the next level. Considering only the effect of the system design on the component, however, could result in prematurely "pigeon-holing" a system based on the initial conceptual design. This can be dangerous given that conceptual system assessment is typically quite approximate (i.e., low fidelity). Using higher-fidelity models at the conceptual level is an alternative, but this tactic is limited by time and resource constraints. Rather, if the design process is to result in an efficient system it must consider feedback from the lower (i.e., component) level designs to the system level. In other words, the system-level design must be reconsidered given additional detail provided by component design. Various models for system design (e.g., the System Engineering Vee, waterfall, and spiral models) depict this common theme of iterative feedback between design levels.[2] Although the concept is prevalent in basic systems engineering theory, this interlevel communication is rarely automated and usually ad hoc at best. For design problems defined via optimization formulations, it is logical to use optimization to synthesize interlevel design levels as well. Only in this way can one ensure optimality at the system level as well as compatibility of designs at lower levels.

One technique for integrating designs across components or disciplines is to formulate the design as a multidisciplinary-optimization (MDO) problem. (This technique is applicable when disciplinary analyses can be represented mathematically.) Efficiency of the overall design can be targeted as the design objective (e.g., minimize weight, minimize cost, etc.), whereas the performance requirements relating to multiple disciplines are treated as constraints. In fact, design optimization is one area cited by industry leaders as an "opportunity for breakthrough" with respect to large-scale design.[3] Several authors have addressed the integrated or multidisciplinary design of components for aerospace systems.[4–7] In particular, Cerro et al. provide a system design hierarchy for the structural sizing of launch vehicle components.[8] To date, MDO formulations for launch vehicles have been at a single design level. In addition, recent research has introduced the concept of approximation and model management optimization, an approach that adaptively increases fidelity of design models (in this case via refining the local resolution of response surfaces) as the design process converges to a final solution.[9] Nevertheless, optimization still occurs at a single design level. The strategy presented in this paper synthesizes two distinct levels of design (i.e., a low-fidelity design at the system level and higher-fidelity design at the component level) as a multidisciplinary optimization problem. This formalizes the feedback communication between levels.

Design at any level requires an assurance of reliability and quality. However, uncertainties in the system characteristics and demand prevent such assurances from being given with absolute certainty. Traditional deterministic design methods have accounted for uncertainties through empirical safety factors. However, such safety factors do not provide a quantitative measure of the safety margin in design and are not quantitatively linked to the influence of different design variables and their uncertainties on the overall system performance. Therefore, in recent decades probabilistic analysis (a rational approach that quantifies the reliability of performance or risk of failure in probabilistic terms and includes these estimates directly in the design optimization) is gaining increased attention. Probabilistic analysis characterizes the uncertainties in a system and uses this information to predict the reliability of achieving a desired response. These uncertainties stem from a number of sources. For example, there is inherent variability such as that produced by the environment (e.g., atmospheric conditions) in which a system operates or variability resulting from manufacturing tolerances. There is also uncertainty that results from incomplete knowledge either about a system itself (as is the case in early stages of design) or how it will behave (e.g., uncertainty in the predictive capability of computational models). Uncertainty from any source can be included in the proposed method as long as it can be modeled with a probability density function.

RBDO techniques have been developed in recent years to achieve design objectives that are stated as optimization problems under uncertainty.[10,11] These techniques have also been applied to aerospace problems.[12] Unfortunately, however, the computational effort required for earlier RBDO methods has been prohibitive for large-scale problems because reliability analyses were nested inside optimization iterations. However, recent advances in decoupling reliability analysis and optimization have led to significant improvements in the computational efficiency of RBDO methods.[13–16]

Thus this paper develops a probabilistic optimization methodology for aerospace vehicle design that takes into account linkages between system-level and component-level design requirements. This methodology formalizes the interlevel feedback coupling required by a systems engineering design approach. The proposed methodology represents the system-component design as a MDO problem. The system design considered optimizes the geometry of a RLV for minimum weight while satisfying aerodynamic constraints. The component design illustrated relates to the structural sizing of vehicle components, in this case a liquid-hydrogen (LH_2) tank. The optimization formulation includes probabilistic constraints, which are evaluated using the limit state-based reliability analysis methodology. The system and component designs are linked through stochastic state variables relating to vehicle geometry and component weight, and the optimization at both levels is pursued. Finally, the reliability-based multidisciplinary optimization is made efficient by decoupling the optimization and reliability analysis iterations.[16]

This paper is organized into seven remaining sections. The first two sections set up an illustrative design problem, presenting the system-level and component-level reliability-based optimizations respectively. The next section describes how the system and component levels are coupled; this is followed by an integrated reliability-based optimization formulation, which takes into account both system and component constraints. The decoupled technique used to solve the reliability-based design optimization is then discussed in some detail. The final two sections discuss results and present conclusions.

System Design (RLV Geometry)

Establishing the rough geometry of the vehicle is a system-level analysis. At this level, it is necessary to approximate component contributions to the design in a low-fidelity or nondetailed manner. In this "sample" case, weight-estimating relationships (WER) developed from vehicles already in the inventory are used for the conceptual sizing of new launch vehicles through the code, CONSIZ.[8,17] These WERs assess component contributions to the overall vehicle weight without getting into detailed analyses such as that required for assessing component structural performance. This analysis of the vehicle weight distribution is input into an initial aerodynamic performance assessment. In addition, it provides a conceptual framework from which to base the more detailed design and analysis of components. The combination of weight prediction and aerodynamic performance assessment is the system-level design considered here. (The system-level design is treated independently in more detail in an earlier work.[18])

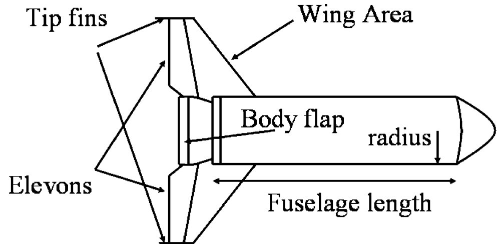

Fig. 1 Illustrative vehicle geometry concept.

Low-fidelity second-order response surface models were developed for a deterministic sizing analysis of a wing-body, single-stage-to-orbit vehicle.[17] For this application, a launch vehicle is sized to deliver a 25,000-lb payload from the Kennedy Space Center to the International Space Station. The vehicle geometry, for illustration purposes, is shown in Fig. 1 and has a slender, round fuselage and a clipped delta wing. Elevons provide aerodynamic and pitch control. Vertical tip fins provide directional control, and body flaps provide additional pitch control.

As a first step in the conceptual design, two analyses (weight prediction and aerodynamics) are considered in a constrained optimization problem. A vehicle geometry that minimizes mean dry weight is expected to minimize overall cost, and so this is chosen as the objective function. For stability, the pitching moment C_m for the vehicle should be zero or extremely close to zero. In addition, C_m should decrease as the angle of attack increases. This is achieved by adjusting the control surfaces to trim the vehicle as the angle of attack is increased. Thus the aerodynamic analysis for pitching moment constrains the optimization. Additional constraints are placed on the lift-to-drag ratio for hypersonic flight (L/D), tail volume coefficient (tvc), and the ratio of landed weight to standard reference wing area and coefficient of lift ($W/S/C_L$). The hypersonic L/D is set to be greater than 1.2 to achieve a desired cross-range capability during entry. $W/S/C_L$ constraint limits the landing speed [227 corresponds to a landing speed of around 200 kn (103 m/s)], and a maximum tvc value of 0.05 is set simply to limit the size of the tail fins.

The optimal vehicle design is determined by six design variables: fineness ratio (fuselage length/radius), wing area ratio (wing area/radius^2), tip fin area ratio (tip fin area/radius^2), body flap area ratio (body flap area/radius^2), ballast weight fraction (ballast weight/vehicle weight), and mass ratio (gross liftoff weight/burnout weight). For the aerodynamic part of the analysis, three additional variables are required to describe the adjustment of control surfaces to trim the vehicle: angle of attack, elevon deflection, and body flap deflection. The pitching-moment constraint must hold during all flight conditions; nine flight scenarios (constructed with three velocity levels and three angles of attack) are used as a representative sample. The representative velocities (Mach 0.3, 2, and 10) were selected as those originally used in Unal et al.[17] for which response surfaces were previously generated. The deterministic optimization problem was written as follows: Minimize vehicle dry weight W_{empty} such that the pitching-moment coefficient C_m for each of nine scenarios is within acceptable bounds $[-0.01, +0.01]$.

The problem was reformulated in probabilistic terms as follows: Minimize mean weight to meet aerodynamic constraints {e.g., such that the pitching-moment coefficient for all nine scenarios has a low probability (less than 0.1) of failing to be within the acceptable bounds $[-0.01, +0.01]$}. Note here that the output parameters (weight W_{empty} and pitching moment C_m) are random variables. They cannot be known exactly because the inputs and analysis model, on which they are based, is subject to uncertainty. Therefore minimizing the mean weight approximates the weight minimization, and the pitching-moment constraint is estimated as a probability of failure to be within acceptable bounds. The solution of this revised, probabilistic problem requires characterization of system uncertainties, defining the limit states for failure, probabilistic analysis, and finally optimization.

System uncertainty comes from various sources and can be modeled through probability density functions for input parameters that are treated as random variables. For example, for the design parameters just mentioned, the as-built conditions might not exactly be the same as the design specifications made at this early conceptual level. Furthermore, uncertainties in operational performance lead to randomness in the control variables.

Constraints can be treated as constraints on the distribution parameters of the design variables that are random (e.g., mean values) or as probability constraints. Here, the first-order mean approximations for tail volume coefficient, hypersonic lift/drag, and relative landed weight/lift constraints are given as constraints. The pitching-moment constraints, however, are formulated in probabilistic terms and evaluated using limit state-based reliability analysis. In this problem, each of the nine scenarios has two pitching-moment limit states, one for the lower bound and one for the upper bound of C_m (pitching-moment coefficient). The lower-bound limit state is

$$g_{\text{lower}} = 0.01 + C_m \tag{1}$$

and the upper-bound limit state is

$$g_{\text{upper}} = 0.01 - C_m \tag{2}$$

The probability of failure is defined as

$$\begin{aligned} P_f &= P(C_m \le -0.01) + P(C_m \ge 0.01) \\ &= P(g_{\text{lower}} \le 0) + P(g_{\text{upper}} \le 0) \end{aligned} \tag{3}$$

A first-order reliability method or FORM[19,20] is used to approximate this probability of failure. Finally, a gradient-based nonlinear optimizer is used to find the minimal mean weight given the probabilistic pitching-moment constraints. Using the probabilistic optimization process just described, the following formulation is solved to obtain the best acceptable geometry, yielding an optimal mean empty weight for the vehicle:

$$\begin{aligned} &\text{Minimize mean of} \quad W_{\text{empty}} \\ \text{Subject to} \quad &P\left(\left|C_{m(i)}\right| \le 0.01\right) \le 0.1, \qquad i = 1 \text{ to } 9 \\ &\text{mean of} \quad tvc \le 0.05 \\ &\text{mean of} \quad W/S/C_L \le 227 \\ &\text{mean of} \quad L/D \ge 1.2 \end{aligned} \tag{4}$$

Component Design (LH_2 Tank Structural Sizing)

As mentioned in the preceding section, the system-level design provides a framework for the more detailed design of individual components. In this case, the weight distribution of the RLV system provides input for inertial loads required for the structural design of individual components.[8,21]

A launch vehicle is composed of many components (Fig. 2). Each component must be designed to successfully perform its individual function but must also integrate or fit into the system as a whole. For the scope of this analysis, an LH_2 tank is considered. It is assumed to be a typical cylindrical tank with given end eccentricity, located at a fixed distance from the end of the vehicle. The tank is to be sized such that it is as light as possible but strong enough to resist stresses induced by inertial loads, internal pressures, and other forces.

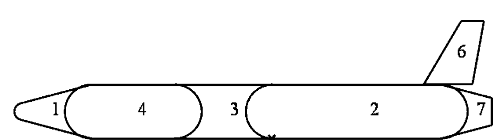

Fig. 2 Launch vehicle components: 1, nose; 2, LH_2 tank; 3, intertank adapter; 4, LOX tank; 5, wing/carry-thru; 6, tail; and 7, thrust structure.

The design goal for the liquid-hydrogen tank then is to minimize the weight of the tank while meeting the requirements for fuel capacity and structural integrity. The fuel capacity requirement is determined by the system-level design (i.e., from weight-estimating relationships used in the system-level weights analysis). At the component level, the fuel capacity is maintained by choosing the appropriate tank geometry. A deterministic optimization problem can be formulated to select the best design for the tank wall structure as follows:

$$\text{Minimize tank weight} = f(R)$$

Subject to

$$R - S < 0 \text{ or } (R/S) - 1 \leq 0 \text{ (for all failure modes)} \tag{5}$$

where R is the tank resistance and S is the loading on the tank. Here R and S are generic symbols for resistance and loading and can be tailored for different failure modes. The left-hand side of the preceding constraint is referred to as a limit state function in reliability analysis literature.[22] The problem is reformulated to consider the uncertainties in R and S:

$$\text{Minimize mean of tank weight} = f(f_R(R)) \approx f(\mu_R)$$

Subject to

$$P(R - S) < P_{\text{required}} \text{ (for all failure modes)} \tag{6}$$

This optimization formulation recognizes that the objective (tank weight) and constraints (failure limit states) are random variables. For well-defined optimization, objectives and constraints need to be selected from among the parameters that characterize the random distributions of these variables. In this case, the parameter mean tank weight is selected as the objective, and the probability of system failure is chosen as the constraint.

There are multiple modes of failure for the tank (i.e., Von Mises interaction failure, isotropic failure, panel buckling), multiple locations along the tank that could fail, and even multiple load cases (at various stages in the vehicle trajectory) that could cause failure. Each of these failure cases can be represented by a corresponding limit state. However, the overall reliability measure for the tank is the system failure probability, which synthesizes all of these modes. This system failure is represented by the union individual limit state failures. Several methods are available for approximating the union or intersection of several events.[23–27] However, to simplify the optimization problem (for this sample problem) representative failure modes are given individual failure probability limits in lieu of a system failure constraint. Evaluating the structural failure criteria then involves four subtasks: 1) defining the system loading S, based on information from the system-level analysis and the mission profile; 2) defining analytical models for various failure modes that incorporate the loading model and resistance R, in terms of design variables; 3) quantifying the uncertainty in the inputs to the failure model; and 4) using probabilistic methods to evaluate the failure probability of the individual limit states.

For the first subtask, system load calculations are based on a simple beam model in this paper, for the sake of illustration. For the second subtask, a multimode failure model of the system is considered. This model synthesizes three failure modes for a honeycomb sandwich wall tank, consisting of 40 individually designed panels (Fig. 3). The honeycomb sandwich consists of top and bottom plates of aluminum, AL2024, and Hexcell 1/8-in.-5052-.0015 for the sandwich material. Design of the panels must specify the thickness of the plates and sandwich. For the tank walls, the significant failure modes are exceeding isotropic strength in the transverse direction, exceeding Von Mises strength, and honeycomb buckling. Three limit state functions (g_{ISO}, g_{VM}, and g_{HCB}, respectively) are defined such that $g_i < 0$ indicates failure by a particular mode i. To facilitate probabilistic optimization, response surfaces for each failure mode were developed from a design of experiments using commercial structural sizing software.

The third subtask requires modeling system uncertainty. As seen from the first subtask, loading is a function of several variables. Plate thickness t_{plate} is the design variable, and honeycomb thickness t_{hc} is an additional resistance variable. All of these have a degree of uncertainty that affects the structural integrity of the component (i.e., whether the LH_2 tank satisfies the three failure criteria). The variables are summarized in Table 1. The first six variables in Table 1 are determined by the mission profile for the launch vehicle. They vary along the flight trajectory and include two reaction locations ($R1$ and $R2$) representing support locations at liftoff, aerodynamic lift points during flight, or wheel locations during landing. Other mission variables are the fuel percentage, horizontal and vertical components of acceleration, and the liquid-oxygen to LH_2 mixture ratio. The system variables are relevant geometry parameters and component weight predictions obtained from the RLV system design.

For the fourth task, a limit-state based methodology (similar to that described for the system problem) is used to evaluate the probability of any single failure mode.

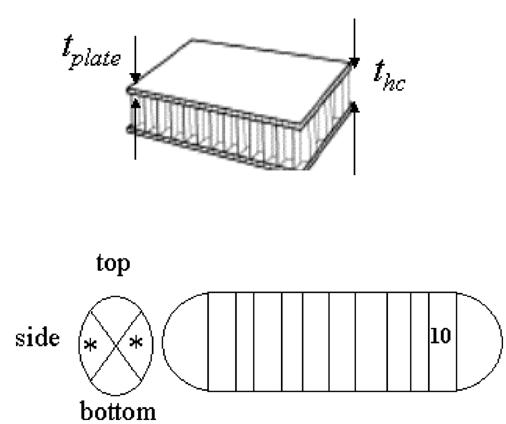

Fig. 3 Segmented honeycomb-wall tank.

Table 1 LH_2 tank sizing variables

Parameter	Origin	Mean	Cov	Description
$R1$	Mission	350 (885.5)	0.1	Location of first reaction point
$R2$	Mission	2000 (5080)	0.1	Location of second reaction point
%_fuel	Mission	0.9	0.1	Percent of fuel remaining in tank
ax	Mission	1 (0.3048)	0.1	Axial acceleration
ay	Mission	1 (0.3048)	0.1	Normal acceleration
Mix ratio	Mission	0.2	0.1	Ratio of LOX weight to LH_2 weight
Radius	System	——	——	RLV and tank radius
Fuel wt	System	——	——	Total fuel weight (LH_2 and lox)
$t$plate	Component	Design var	0.1	Top and bottom plate thickness
$t$hc	Component	0.1 (0.254)	0.1	Honeycomb sandwich thickness
oal	System	——	——	Overall length
wstruct	System	——	——	Distributed load along entire RLV
wwing	System	——	——	Distributed load along wing

The problem statement in Eq. (6) can now be rewritten as follows:

$$\text{Minimize mean of} \quad t_{\text{plate}}$$

$$\text{Subject to}$$

$$P(g_{\text{VM}} \le 0) \le P_{\text{VM acceptable}}, \qquad P(g_{\text{ISO}} \le 0) \le P_{\text{ISO acceptable}}$$

$$P(g_{\text{HCB}} \le 0) \le P_{\text{HCB acceptable}} \tag{7}$$

As with the system design problem, the optimization is performed using a gradient-based nonlinear optimizer.

Data Coupling Between System and Component

As is apparent from Table 1, the component-level design relies on input from the system design. For example, the tank geometry is constrained by the vehicle geometry (the tank radius must be smaller than the vehicle radius) and by the volume of fuel needed for the mission (i.e., propulsion weight). The loads placed on the tank are a function of both vehicle geometry (radius and length) and weight distribution (modeled as uniform distributions for major components). This data flow represents the decomposition phase of design.

After component design is completed, a more accurate estimate of the tank weight is available from Eq. (8) as

$$\sum_{\text{all panels}} (\text{panel length} * \text{panel width}) * (\rho_{\text{plate}} * t_{\text{plate}} + \rho_{\text{hc}} * t_{\text{hc}}) \tag{8}$$

where panel length and width are calculations performed during the structural sizing analysis. The "refined" tank weight can be fed back into the system design to verify if the system-level requirements are still met. Recall that the system design analysis initially accounts for the component weight contributions through the WER. One option is to replace the LH_2 WER with the weight from the component-level analysis as a constant. However, we chose instead to adjust a reduction factor (included as part of the WER) so that the tank weight will adjust as system-level design changes are made. This reduction factor is denoted $\text{rf}_{\text{tank weight}}$ and is updated according to the following formula:

$$\text{rf}_{\text{tank weight}} = 1 - \frac{\text{tank weight from structural sizing}}{\text{baseline tank weight from weights analysis}} \tag{9}$$

where the baseline tank weight is given by a response surface of the LH_2 tank weight (before applying the reduction factor) from the software code CONSIZ.[8,17]

When desiring a true optimal design, a single pass of information from system to component and back is inadequate. Instead an iterative process is needed to converge on optimal solutions for both the system and component designs. Perhaps the most obvious iteration strategy is to use a brute-force fixed-point iteration method; in other words, to simply repeat the system–component–system design cycle and hope for ultimate convergence so that neither design changes in subsequent cycles. This idea is depicted in Fig. 4.

This bilevel optimization is a common strategy for design; it does not require interlevel data flow during optimization and preserves a degree of autonomy for component-level designers. However, this strategy might not be able to find a converged solution to the bilevel system with a reasonable amount of computational effort. As more components are added, finding a feasible solution will become even more difficult.

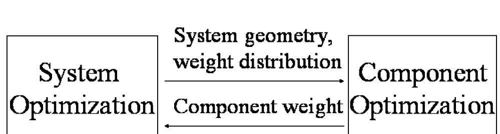

Fig. 4 Fixed-point iteration between system and component-level optimizations.

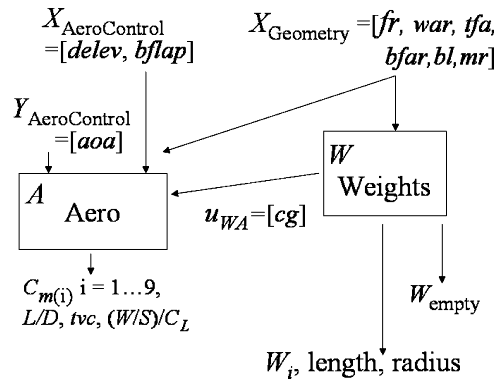

Fig. 5a System design optimization data flow.

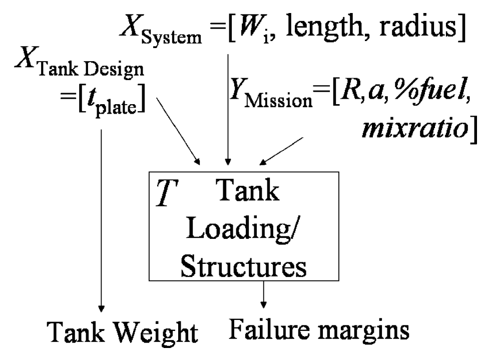

Fig. 5b LH_2 tank design optimization data flow.

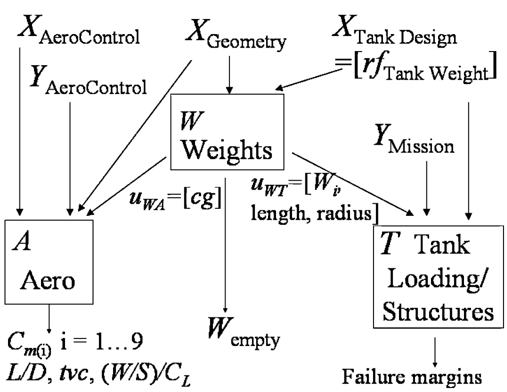

Fig. 6 Integrated multidisciplinary system.

An alternate approach is to integrate the two optimizations through a MDO formulation. To understand how this can be done, it is helpful first to map out the data flow for each design level. Figures 5a and 5b provide such a mapping for the system and component-level optimizations respectively.

(Note that in Fig. 5 design variables are denoted by X, while Y denotes other uncertain input variables.) Figures 5a and 5b reveal several issues. First, from the two-discipline system analysis in Fig. 5a we notice that the quantities of interest for the component tank analysis T come from the weights analysis W; the aerodynamics analysis A is needed only to evaluate the system-level constraints. Similarly, during system updating, the tank weight is directly relevant only to the weights analysis W but affects the aerodynamics discipline through the centers of gravity passed as a state (or coupling) variable from W. In addition, it is evident that to truly couple the system and component analyses, the weights analysis needs modification to make the LH_2 tank weight reduction factor an explicit input. This requires a new design of experiments to generate new weight response surfaces incorporating the additional input. Finally, we propose the multidisciplinary system of Fig. 6 for the system-component coupling. Note in this figure that the tank design variables $X_{\text{TankDesign}}$ from Fig. 5b (plate thickness and honeycomb thickness) have been replaced by the tank weight reduction factor $\text{rf}_{\text{tank weight}}$. This exchange can easily be made because the reduction factor is uniquely determined by the tank design (i.e., plate thickness for given honeycomb thickness) by using Eqs. (8) and (9).

Probabilistic MDO Formulation of RLV System/Tank Design

The influences of uncertainty in the combined design have to be treated carefully. Uncertainty in the geometry propagates through the weights analysis to the other disciplines through the intermediate state variables (e.g., c.g. the weight distribution, length, radius, and tank weight). These uncertainties combine with uncertainty in the aerocontrol and mission variables resulting in uncertainty in the aerodynamic constraints as well as in the structural failure analysis. With this uncertainty propagation in mind, an optimization formulation for the system given in Fig. 6 follows:

$$\text{Minimize mean of} \quad W_{\text{empty}}$$

$$\mu_{X\text{Geomery}},\ \mu_{rf},\ \mu_{X\text{aeroControl}}$$

$$\text{subject to}$$

$$P\left(\left|C_{m(i)}\right| \leq 0.01\right) \leq P_{\text{acceptable}} \qquad \text{for} \qquad i = 1\ldots 9$$

$$P(g_{\text{VM}} \leq 0) \leq P_{\text{acceptable}}, \qquad P(g_{\text{ISO}} \leq 0) \leq P_{\text{acceptable}}$$

$$P(g_{\text{HCB}} \leq 0) \leq P_{\text{acceptable}}$$

$$\text{mean of} \qquad tvc \leq 0.05$$

$$\text{mean of} \qquad W/S/C_L \leq 227$$

$$\text{mean of} \qquad L/D \geq 1.2$$

$$\text{where} \qquad C_{m(i)} = A(X_{\text{Geometry}}, X_{\text{AeroControl}}, u_{\text{WA}}, Y_{\text{AeroControl}}) \qquad i = 1\ldots 9$$

$$g_j = T(u_{\text{WT}}, Y_{\text{Mission}}), \qquad j = \text{VM, ISO, HCB} \tag{10}$$

The mean values (denoted by μ) of the input variables (X_{Geometry}, $X_{\text{AeroControl}}$, and $\text{rf}_{\text{tank weight}}$) are the design variables. The first-order mean approximation for empty weight is the objective just as in the system-level analysis. Probabilistic pitching-moment constraints are also used as in the system-level analysis; they are functions of geometry inputs, aerodynamic control inputs, the coupling variable, u_{WA} (i.e., center of gravity) from the weight analysis, and the random parameters ($Y_{\text{AeroControl}}$). Similarly, first-order mean values for tvc, $W/S/C_L$, and L/D are given as constraints. The probabilistic constraints for structural failure are the same as those given in Eq. (7), specifically the probability of three significant modes of failure dependent on output from the weights analysis u_{WT} and the random parameter Y_{Mission}. The structural sizing objective (i.e., minimize tank plate thickness) disappears from the formulation. However, because the plate thickness directly affects the vehicle weight, minimizing the overall objective (i.e., vehicle empty weight) will also ensure minimal tank plate thickness.

Conveniently, the probabilistic MDO formulation in Eq. (10) does not have feedback coupling. In other words, with the integration of the two problems the individual analyses are dependent in a single direction; the aerodynamic and structural analyses depend on data flow from the weights analysis, but the weights analysis does not require input from the other disciplines. For example, in Eq. (10) the pitching moment C_m is written as a function of X_{Geometry}, $X_{\text{AeroConstrol}}$, u_{WA}, and the aerodynamic analysis A. However, u_{WA} is determined by the weights analysis W so that we can represent the sequence of analyses (weights analysis followed by aerodynamic analysis) as $W - A$. $W - A$ is a function of all inputs to W and A (i.e., X_{Geometry}, $X_{\text{AeroControl}}$, and $rf_{\text{tank weight}}$), but the coupling variable u_{WA} does not need to appear explicitly. Similarly, if we represent the weights analysis followed by the tank sizing analysis as $W - T$, the coupling variable u_{WT} need not appear. As an example, the function for pitching moment is given in Eq. (11) without the coupling variable u_{WA}:

$$C_{m(i)} = W - A(X_{\text{Geometry}}, X_{\text{AeroControl}}, rf_{\text{tank weight}}, Y_{\text{AeroControl}}) \qquad i = 1\ldots 9 \tag{11}$$

It is important to realize that the absence of feedback coupling is only a feature of this particular problem and might not necessarily be the case for system to component integration problems in general.

Solution Strategy: Decoupled RBDO

We solve the optimization using a decoupled RBDO technique. This technique was proposed by Du and Chen as an integral part of their sequential optimization and reliability analysis method (SORA).[16] (The SORA method includes other improvements aimed at reducing computational effort. However, only the basic decoupling technique is implemented in this paper.) During traditional (fully coupled) reliability-based optimization, probabilistic analysis is required during each iteration of the optimizer to evaluate the probabilistic constraints. (Additional probabilistic analysis might also be required to evaluate the derivatives of the probabilistic constraints when gradient-based optimization algorithms are used.) Probabilistic analysis is also an iterative process, so that traditional reliability-based optimization is a nested loop analysis (i.e., a probabilistic analysis loop nested within optimization). This methodology requires significant computational effort. The decoupling method avoids this nesting of reliability analysis within the optimization. Instead, optimization is performed using a deterministic equivalent for the probabilistic constraint. Reliability analysis follows optimization sequentially as opposed to being integrated within the optimization. Here the reliability analysis provides an update to the deterministic equivalent of the probabilistic constraint; optimization using the new constraint follows, and the process is repeated until convergence.

The reliability analysis used in the decoupled RBDO in this paper is based on the performance measure approach (PMA).[13] PMA is the inverse of the reliability index approach (RIA),[28] the traditional approach at implementing the FORM. Consider a generic probabilistic constraint:

$$P[g(\boldsymbol{x}) \leq 0] \leq P_{\text{acceptable}} \tag{12}$$

where $\boldsymbol{x}$ is a vector of random design variables. By using a first-order approximation, this constraint can also be represented in terms of the reliability index β:

$$P[g(\boldsymbol{x}) \leq 0] \approx \Phi(-\beta) \leq \Phi(-\beta_{\text{target}}) \tag{13}$$

or equivalently

$$\beta \geq \beta_{\text{target}}$$

In RIA β is obtained from the following minimization:

$$\text{Min}\, \beta = \|\eta\|_2$$

$$\text{subject to}$$

$$g(\boldsymbol{x}) = 0 \tag{14}$$

where $\boldsymbol{\eta}$ is the vector of random input variables in standard normal space given by the transformation in Eq. (15) for each random variable x_i in $\boldsymbol{x}$. The terms μ_x^N and σ_x^N are the equivalent normal mean value and standard normal standard deviation, respectively:

$$\eta = \left(x - \mu_x^N\right) \big/ \sigma_x^N \tag{15}$$

The value of $\boldsymbol{x}$ that satisfies Eq. (14) is known as the most probable point. The computation in Eq. (14) is depicted graphically in two dimensions in Fig. 7.

With the PMA approach, this same probabilistic constraint [Eq. (12)] is enforced in the inverse manner. Constraint satisfaction is ensured when the minimum value of the limit state g for a

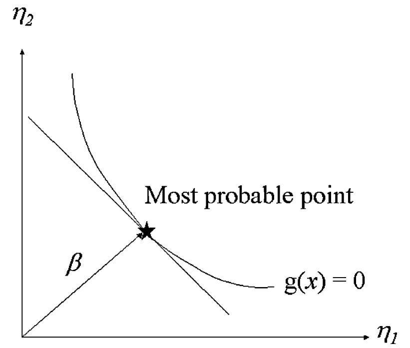

Fig. 7 Reliability index method.

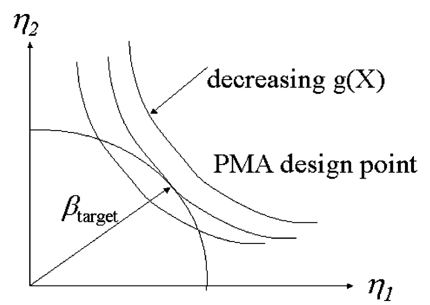

Fig. 8 First-order performance measure approach.

target reliability index β_{target} is greater than zero. So we write the following constraint:

$$g(\boldsymbol{x}^*) \geq 0 \tag{16}$$

where the PMA design point $\boldsymbol{x}^*$ is the solution to the minimization problem in Eq. (17):

$$\text{Min}\, g(\boldsymbol{x})$$

$$\text{subject to}$$

$$\|\eta\|_2 = \beta_{\text{target}} \tag{17}$$

Again, the standard normal transformation for η is given by Eq. (15). A graphical representation of PMA in two dimensions is given in Fig. 8. From this figure, one can see that if Eq. (17) is satisfied for the PMA design point the most probable point of the RIA method will lie inside or on the target β circle so that the reliability index β will be less than or equal to β_{target}. Thus the PMA constraint given by Eqs. (16) and (17) and the RIA constraint given in Eqs. (13) and (14) are equivalent.

This decoupled RBDO method used here takes advantage of the PMA method in order to decouple the optimization and reliability analysis by separating the random design variables into a deterministic design component used for optimization and a stochastic component used for the reliability analysis. Rearranging the standard normal transformation in Eq. (15) gives a particular realization for a random variable x:

$$x_i = \sigma_x^N * \eta_i + \mu_x^N \tag{18}$$

(Note that for non-Gaussian variables, a transformation is required to obtain the equivalent normal mean μ_x^N and standard deviation σ_x^N from the distributions.[22]) The design (i.e., the optimization) determines the values of the distribution parameters: mean μ_x and standard deviation σ_x. In this case, we want to control only the mean values; thus, the design variable is μ_x, and the standard deviation σ_x is dependent on the mean value. Note that it is not possible to write the RIA constraint ($\beta \geq \beta_{\text{target}}$) as a function of both the deterministic and stochastic components of the random variables. (β is a function of only the standard normal realization η.) However, we can rewrite the PMA constraint [Eq. (16)] as a function of both the mean value and the standard normal realization η as

$$g\left(\mu_x, \eta^*\right) \geq 0 \tag{19}$$

The particular standard normal realization η^* satisfies the minimization in Eq. (20), which is simply Eq. (14) rewritten with $\boldsymbol{x}$ decomposed into μ_x and η:

$$\text{Min}\, g\left(\mu_x, \eta^*\right)$$

$$\text{subject to}$$

$$\|\eta\|_2 = \beta_{\text{target}} \tag{20}$$

With this decoupled representation, the optimization is implemented independent of probabilistic analysis by controlling only the deterministic component of the random design variable (i.e., μ_x). During this phase, the stochastic component η is kept constant. (An initial value η^0 is assumed. Future values η^{k-1} are computed during the probabilistic analysis phase.) With η constant, the constraint given by Eq. (19) is now deterministic, and deterministic optimization is possible. A candidate optimal design μ_x^k results for the kth optimization. This value of μ_x^k is fed to the next phase, probabilistic analysis. During this phase, μ_x^k is kept constant, and the value of η satisfying Eq. (20) is found. This resulting η^k is then forwarded to update the constraint (19) for the next optimization $k+1$. The algorithm terminates when successive cycles converge to consistent values of μ_x^k and η^k.

This method can be implemented for the problem given in Eq. (11). In this case, the deterministic optimization phase solves the following formulation:

$$\underset{\mu_{X\text{Geometry}},\, \mu_{rf},\, \mu_{X\text{aeroControl}}}{\text{Minimize mean of}} \quad W_{\text{empty}} \approx W(\mu_{X\text{Geometry}}, \mu_{rf})$$

$$\text{subject to}$$

$$\left|C_{m(i)}\right| = W - A\left[\mu, \eta^{k-1(i)}\right] \geq 0 \qquad \text{for} \qquad i = 1 \ldots 9$$

$$g_{\text{VM}} = W - T\left[\mu, \eta^{k-1(10)}\right] \geq 0$$

$$g_{\text{ISO}} = W - T\left[\mu, \eta^{k-1(11)}\right] \geq 0$$

$$g_{\text{HCB}} = W - T\left[\mu, \eta^{k-1(12)}\right] \geq 0$$

$$\text{mean of} \quad tvc \leq 0.05$$

$$\text{mean of} \quad W/S/C_L \leq 227$$

$$\text{mean of} \quad L/D \geq 1.2 \tag{21}$$

Here μ represents the mean values for all random variables (i.e., $\mu_{X\text{Geometry}}$, $\mu_{X\text{AeroControl}}$, μ_{rf}, $\mu_{Y\text{Mission}}$, and $\mu_{Y\text{AeroControl}}$). The parametric constraints (mean of tvc, mean of $W/S/C_L$, and mean of L/D) are already in deterministic form so that they appear exactly as in the original formulation. However, each of the probabilistic constraints have been replaced by a deterministic equivalent in the form of Eq. (16). These constraints are functions of the parametric design quantities (mean values of X_{Geometry}, $X_{\text{AeroControl}}$, and $rf_{\text{tank weight}}$) and the stochastic components of all random variables η^k.

Following the optimization, a PMA analysis is required for each probabilistic constraint to determine $\eta^{k(i)}$. For example, for a single pitching-moment constraint PMA finds the η that solves the following minimization for constant mean values:

$$\underset{\eta}{\text{Min}}\left|C_{m(i)}\right|$$

$$= W - A\left(\mu_{X(\text{Geometry})}^k, \mu_{X(\text{AeroControl})}^k, \mu_{rf}^k, \mu_{Y(\text{AeroControl})}, \eta\right)$$

subject to

$$\beta = ||\eta||_2 = -\Phi^{-1}(0.1) \tag{22}$$

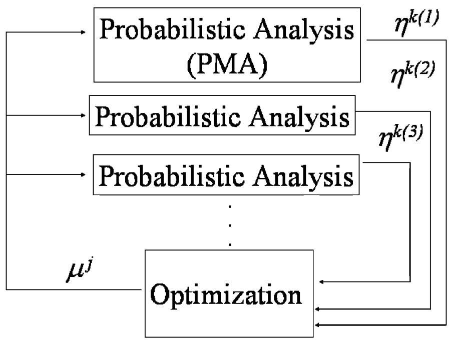

Fig. 9 Decoupled RBDO for multiple constraints.

The resulting random realization η then becomes $\eta^{k(i)}$ for the next optimization. Note that each constraint is associated with its own $\eta^{k(i)}$ because $\eta^{k(i)}$ represents the current estimate of the PMA design point for that particular constraint i. This concept for multiple constraints is outlined graphically in Fig. 9.

Results and Discussion

The optimization [Eq. 10)] was solved by using a MATLAB® routine, which implements the reliability-based optimization (as just outlined) with a sequential-quadratic-programming algorithm from their Optimization Toolbox.[29] A converged solution was obtained in four iterations. (For comparison, a traditional nested RBDO approach based on the reliability index was attempted but was not able to converge to a solution.) The results are given in Tables 2–4 for two different reliability constraints: a 10% probability of constraint failure and a 0.0013 probability of constraint failure (corresponding to target reliability indices of 1.28 and 3, respectively). As expected, the higher reliability requirement results in a larger mean vehicle weight (about a 5% increase in mean weight for a slightly less than 100-fold improvement in reliability). For both reliability levels, the two active constraints are the ninth pitching-moment constraint (corresponding to a maximum angle of attack at hypersonic speed) and the isotropic strength constraint. A postoptimization sensitivity analysis (based on relative partial derivatives at the design point) reveals that the most significant variables for empty weight are the fineness and wing area ratios. These two variables are also the most significant for the pitching-moment constraints. However, the upper-bound constraint for the mean of the fineness ratio is also active, limiting its contribution to improve the optimal weight. The probabilistic constraint for isotropic strength failure is dominated by the tank weight reduction factor $rf_{\text{tank weight}}$.

Note that the function evaluations for each RBDO phase (i.e., the optimization phase and the probabilistic analysis phase) include evaluations required for finite difference approximations of the gradient. In the optimization phase, there are 15 constraints and 22 design variables, so that a minimum of 331 function evaluations are required to approximate the Jacobian during each iteration of the optimizer. In this case, each optimization phase requires approximately 4000 function evaluations in an average of 12 iterations. (Note that a deterministic safety-factor-based formulation of the original problem would be also be expected to require about 4000 function evaluations using finite differences to approximate the gradient.) In the probabilistic analysis phase, there are 38 random variables so that a minimum of 39 function evaluations is required for every probabilistic analysis loop; this is required for each of the 12 probabilistic constraints. The importance of derivative calculations in large-scale optimization problems is well documented,[2] and these results only reinforce their significance. In this example, there would be considerable value in identifying inactive constraints so that those calculations could be avoided. This was not done, but in hindsight could have resulted in a more than sevenfold improvement in computational effort given that only two of the 15 constraints were active at the optimum. Even so, the total computational effort for the decoupled RBDO (e.g., 26,121 function evaluations for a 10% failure probability) is under seven times that for deterministic optimization (roughly 4000 function evaluations).

Table 2 Design requirements

Variable	Case 1	Case 2
P_f	0.10	0.0013
β_{target}	1.28	3.00

Table 3 Optimization results

		Optimal design	
Bounds	Design variable	Case 1	Case 2
[0, 0.9]	$\text{rf}_{\text{tank weight}}$	0.033	0.00
[4, 7]	fr	7.00	7.00
[10, 20]	war	15.99	16.839
[0.05, 3.0]	tfar	0.50	0.96
[0, 0]	bfar	0.00	0.00
[0, 0.4]	bl	0.0033	0.0013
[7.5, 8.25]	mr	7.74	7.76
Objective	Empty weight, lb (kg)	202,180 (91,709)	212,800 (96,526)

Table 4 Computational effort

Parameter	Case 1	Case 2
Decoupled RBDO iterations	4	4
Optimization function evaluations	15,349	13,932
Probabilistic analysis function evaluations	10,772	10,358

Another significant observation is that the total number of evaluations does not increase as the required failure probability is decreased. This is an advantage of using an analytical approach (i.e., first-order reliability analysis) to evaluating the probabilistic constraint as opposed to Monte Carlo simulation-based methods. However, accuracy of the reliability estimate could be compromised, especially for highly nonlinear constraints. Therefore, the final design should be checked with Monte Carlo simulation.

Conclusions

Design by decomposition is a fairly common and practical strategy for complex engineering. However, some degree of integration is required to ensure the multilevel designs are compatible. This is a special challenge when the effects of uncertainty are considered. The process outlined in this paper presents a strategy for coupling design levels as a multidisciplinary optimization under uncertainty.

The example application demonstrates some obvious advantages and drawbacks for this approach. First, by using reliability-based design optimization (RBDO), reliability requirements can be explicitly enforced during design. A deterministic factor of safety design, on the other hand, does not provide a quantitative measure of reliability. The RBDO approach also allows engineers to see the effects of varying reliability requirements on design optimality, which is extremely useful in making informed tradeoff decisions. By using a multidisciplinary optimization to couple design levels, the uncertainty information also passes formally between system-and component-level designs. This approach prevents low-fidelity system-level analyses from unduly restricting future component-level design decisions. The obvious drawback for the methodology is the increase in computational effort over deterministic methods. However, decoupled RBDO methods reduce this liability significantly.

Incorporating the design of additional components would require additional probabilistic constraints and additional design variables linking component requirements to the system-level objectives (e.g., reduction factors for weights from each component). The multidisciplinary-optimization problem complexity and the computational effort required to solve it will increase proportionally. However, this approach is likely less difficult than attempting to integrate the individual component designs directly with one another on a single level. Another added complexity would be to consider additional component design variables (e.g., tank properties other than plate thickness). In this case, it might not be possible to use a variable such as the tank weight reduction factor to link the system and component weight analyses. Instead, a component-level optimization could be used for the structural sizing analysis of the tank. Finally, the issue of how to handle system reliability constraints (such as a system failure defined by the union of several failures) in conjunction with efficient reliability-based optimization needs to be addressed.

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References

[1]"2nd Generation RLV System Engineering and Risk Reduction," NASA NRA8-30, Oct. 2000.

[2]Buede, D., *The Engineering Design of Systems*, Wiley, New York, 2000, Chap. 1.

[3]Alexandrov, N. M., Alter, S. J., Atkins, H. L., Bey, K. S., Bibb, K. L., Biedron, R. T., Carpenter, M. H., Cheatwood, F. M., Drummond, P. J., Gnoffo, P. A., Jones, W. T., Kleb, W. L., Lee-Rausch, E. M., Merski, N. R., Mineck, R. E., Nielsen, E. J., Park, M. A., Pirzadeh, S. Z., Roberts, T. W., Samareh, J. A., Swanson, R. C., Thomas, J. L., Vasta, V. N., Weilmuenster, K. J., White, J. A., Wood, W. A., and Yip, L. P., "Opportunities for Breakthroughs in Large-Scale Computational Simulation and Design," NASA TM-2002-211747, June 2002.

[4]Braun, R. D., Powell, R. W., Lepsch, R. A., and Stanley, D. O., "Comparison of Two Multidisciplinary Optimization Strategies for Launch Vehicle Design," *Journal of Spacecraft and Rockets*, Vol. 32 No. 3, 1995, pp. 404–410.

[5]Unal, R., Morris, W. D., White, N. H., and Lepsch, R. A., "Modeling and Simulation of Reliability and Maintainability Parameters for Reusable Launch Vehicles Using Design of Experiments," *19th American Society for Engineering Management Conference*, ASEM, Rolla, MO, Oct. 1998, pp. 90–94.

[6]Tartabini, P. V., Lepsch, R. A., Korte, J. J., and Wurster, K. E., "A Multidisciplinary Performance Analysis of a Lifting-Body Single-Stage-to-Orbit Vehicle," AIAA Paper 2000-1048, 2000.

[7]Korte, J. J., Weston, R. P., and Zang, T. A., "Multidisciplinary Optimization Methods for Preliminary Design," Paper C-40, AGARD, April 1997.

[8]Cerro, J. A., Martinovic, Z. N., Su, P., and Eldred, L., "Structural Weight Estimation for Launch Vehicles," *Proceedings of the 61st Annual International Conference on Mass Properties Engineering*, Paper 3201, SAWE, Los Angeles, 2002.

[9]Alexandrov, N. M., Lewis, R. M., Gumbert, C. R., Green, L. L., and Newman, P. A., "Optimization with Variable-Fidelity Models Applied to Wing Design," AIAA Paper 2000-0841, 2000.

[10]Mahadevan, S., "Probabilistic Optimum Design of Framed Structures," *Computers and Structures*, Vol. 32, No. 3, 1992, pp. 365–374.

[11]Pu, Y., Das, P. K., and Faulkner, D., "A Strategy for Reliability-Based Optimization," *Engineering Structures*, Vol. 19, No. 3, 1997, pp. 276–282.

[12]Padula, S., Gumbert, C., and Li, W., "Aerospace Applications of Optimization Under Uncertainty," *Proceedings from the 4th International Symposium on Uncertainty Modeling and Analysis*, IEEE, Washington, DC, 2003, pp. 286–291.

[13]Tu, J., Choi, K. K., and Park, Y. H., "A New Study on Reliability-Based Design Optimization," *Journal of Mechanical Design*, Vol. 121, No. 4, 1999, pp. 557–564.

[14]Royset, J. O., Der Kiureghian, A., and Polak, E., "Reliability-Based Optimal Structural Design by the Decoupling Approach," *Reliability Engineering and System Safety*, Vol. 73, No. 3, 2001, pp. 213–221.

[15]Wu, Y. T., Shin, Y., Sues, R., and Cesare, M., "Safety Factor Based Approach for Probabilistic-Based Design Optimization," AIAA Paper 2001-1522, 2001.

[16]Du, X., and Chen, W., "Sequential Optimization and Reliability Assessment Method for Efficient Probabilistic Design," DETC2002/DAC-34127, ASME, New York, Sept.–Oct. 2002.

[17]Unal, R., Lepsch, R. A., and McMillin, M. L., "Response Surface Model Building and Multi-Disciplinary Optimization Using D-Optimal Designs," *Proceedings of the 7th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, Vol. 1, AIAA, Reston, VA, 1998, pp. 405–411.

[18]Smith, N., and Mahadevan, S., "Probabilistic Methods for Aerospace System Conceptual Design," *Journal of Spacecraft and Rockets*, Vol. 40, No. 3, 2003, pp. 411–418.

[19]Hasofer, A. M., and Lind, N. C., "Exact and Invariant Second Moment Code Format," *Journal of the Engineering Mechanics Division*, Vol. 110, No. EM1, 1974, pp. 111–121.

[20]Rackwitz, R., and Fiessler, B., "Structural Reliability Under Combined Random Load Sequences," *Computers and Structures*, Vol. 9, No. 5, 1978, pp. 484–494.

[21]Mahadevan, S., and Smith, N., "System Risk Assessment and Allocation in Conceptual Design," NASA CR-2003-212162, May 2003.

[22]Haldar, A., and Mahadevan, S., *Probability, Reliability and Statistical Methods in Engineering Design*, 1st ed., Wiley, New York, 2000, Chap. 7.

[23]Ditlevsen, O., "Generalized Second Moment Reliability Index," *Journal of Structural Mechanics*, Vol. 7, No. 4, 1979, pp. 435–451.

[24]Hohenbichler, M., and Rackwitz, R., "First-Order Concepts in System Reliability," *Structural Safety*, Vol. 1, No. 3, 1983, pp. 177–188.

[25]Gollwitzer, S., and Rackwitz, R., "An Efficient Numerical Solution to the Multinormal Integral," *Probabilistic Engineering Mechanics*, Vol. 3, No. 2, 1988, pp. 98–101.

[26]Madsen, H. O., Krenk, S., and Lind, N. C., *Methods of Structural Safety*, Prentice–Hall, Upper Saddle River, NJ, 1986.

[27]Xiao, Q., and Mahadevan, S., "Second-Order Upper Bounds on Probability of Intersection of Failure Events," *Journal of Engineering Mechanics*, Vol. 13, No. 4, 1994, pp. 207–226.

[28]Yu, X., Choi, K. K., and Park, Y. H., "A Mixed Design Approach for Probabilistic Structural Durability," *Journal of Structural Optimization*, Vol. 14, No. 2–3, 1997, pp. 81–90.

[29]"Optimization Toolbox User's Guide," Ver. 2, MathWorks, Inc., Natick, MA, 2003.

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